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Similarity Criteria for Manipulator Loading and Control Sensitivity Characteristics

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On the basis of similarity-theory methods and the authors' previously developed theoretical approach, similarity criteria are developed for the handling qualities of aircraft with different manipulators and control sensitivity characteristics. The basic concepts are presented for the theoretical approach to estimating the influence of various manipulator and control sensitivity characteristics on handling qualities. The method being substantiated allows estimation of the handling qualities of an aircraft with certain manipulator and control sensitivity characteristics by comparing them with the respective data obtained for the aircraft with other characteristics. Recommendations are given for modeling, on ground-based and in-flight simulators, the controllability of an aircraft with certain manipulators and control sensitivity that may differ from those of simulators. The dynamic characteristics required of the manipulator feel systems used in ground-based and in-flight simulators are considered.

Nomenclature

A_* = characteristic amplitude of controlled parameter
$A_{\delta m}$ = manipulator displacement amplitude, mm
$c(t)$ = controlled parameter, $c = \theta, n_z, \phi, \ldots$
d, f, k = equation constants
F = characteristic value of control force applied by a pilot, N
F_{br} = breakout force, N
F_{fr} = friction force, N
F_m = manipulator force, N
F_{n_z} = longitudinal manipulator force per unit normal acceleration, N/g
F_r = generalized control sensitivity: manipulator force per unit normal acceleration, roll acceleration or rate, etc., $F_r = F_{n_z}, F_{\delta m}/L_{\delta_a}, \ldots$
F_v = current value of force felt by a pilot, N
$F_{\delta m}$ = manipulator force gradient, N/mm
$F_{\dot{\delta} m}$ = manipulator damping coefficient, N/(mm/s)$^{-1}$
F_* = desirable value of control force applied by a pilot, N
G = range of permissible values of manipulator loading and control sensitivity characteristics
g = acceleration due to gravity, m/s^2
J = cost function
K = aircraft gain
L_{δ_a} = lateral control sensitivity, rad/(s$^2\cdot$ mm)
m = manipulator inertia, kg
n_z = aircraft normal acceleration, g
PR = pilot ratings, Cooper–Harper scale
q = pitch rate, rad/s
s = Laplace operator
T_a = actuator time constant, s
T_c = computer sampling time, s
t = time, s
V_0 = characteristic velocity, m/s
Y_a = actuator transfer function
Y_{cp} = computer transfer function
$Y_{c/\delta m}$ = manipulator displacement-to-controlled-parameter transfer function
Δ = increment
δ = characteristic value of manipulator displacement felt by a pilot, mm
δ_m = manipulator displacement, mm
δ_v = current value of displacement felt by a pilot, mm
δ_* = desirable value of manipulator displacement felt by a pilot, mm
φ = phase, deg
χ, ρ, ξ = similarity criteria
ω_* = characteristic frequency, rad/s

Subscripts

opt = optimum value
1, 2 = numbering

Symbol

$(\bar{\ })$ = nondimensional value

I. Introduction

CHARACTERISTICS of manipulator loading ($F_{\delta m}, F_{\mathrm{br}}, \ldots$) and control sensitivity ($F_{n_z}, L_{\delta_a}, \ldots$) play an important role in aircraft controllability. However, choosing values for these characteristics presents considerable difficulty in aircraft design. In flight tests and even in ground-based simulator experiments, it is impossible to study in detail the influence of all manipulator and control sensitivity characteristics on handling qualities. (They depend in a complicated way on each other, on aircraft dynamic performance, and on piloting tasks; besides, the possibilities of flight simulators are restricted.) As mentioned in many publications,[1–6] a theoretical approach had not yet been sufficiently developed.

Recently, however, a theoretical approach to estimating the influence of manipulator loading and control sensitivity characteristics on aircraft handling qualities was proposed.[1–3] On the basis of this approach, simple and efficient criteria have been developed that allow estimation of these characteristics' optimum values with regard to their interplay and dependence on aircraft dynamic performance. The validity of the criteria and the approach has been tested in practice during development of Russian airplanes.

The approach is commonly used for different aircraft, manipulators, and control axes because inherent psychophysiological characteristics of pilots constitute its basis, being independent of aircraft class or manipulator type. Yet, certain parameters in the mathematical expressions may depend on the aircraft type and the manipulator. In Refs. 1–3, the values of the parameters were determined mainly for class III aircraft with different manipulators, by comparing

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estimated and experimental data. Determining the parameter values for other aircraft types would demand a considerable amount of experimental work.

The present work aims mainly at developing the similarity criteria on the basis of similarity theory and the theoretical approach. These criteria may allow us to do without or to use just a few of the parameters that depend on aircraft type and manipulator. Namely, these criteria may allow us 1) to estimate the aircraft handling qualities by comparing them with the respective data for the aircraft with different manipulator and control sensitivity characteristics; 2) to compare more accurately the experimental controllability data obtained under different conditions and to generalize the conclusions; and 3) to improve the similarity of handling qualities while modeling an aircraft on ground-based or in-flight simulators because their manipulator and control-sensitivity characteristics may differ to a certain degree, e.g., manipulator characteristics of in-flight or ground-based simulators are invariable.

In Sec. II, the basic concepts of the proposed theoretical approach are described in general because the notions underlying the theory are obvious. However, the theoretical approach as a whole (as a certain system of related notions) does not seem to agree absolutely with the experimental data, being an approximation, which is true of any other theory applied to analysis of aircraft handling qualities, e.g., Refs. 6 and 7. Thus, in Sec. III, in a discussion of the relevant similarity criteria, the theoretical approach is substantiated by comparing estimated and experimental data.

Section IV deals with some dynamic characteristics required of feel systems used in in-flight and ground-based simulators.

II. Theoretical Approach

This approach is based on the three main principles. They are the result of a generalization of inherent qualities of a pilot being a link in a control loop.

A. Basic Concepts

The first principle is based mainly on the psychophysiological aspects of a pilot's assessment of manipulator control forces and displacements.

The principle states that for any pilot there is a certain desirable (optimum) combination of forces and displacements (F_*, δ_*). Deviation of the forces and displacements applied by a pilot (F, δ) from their desirable values (F_*, δ_*) worsens aircraft handling qualities, their being dependent on F/F_*, δ/δ_* only and independent of other factors such as manipulator type, aircraft class, and its dynamics, etc.

If an aircraft has good pilot ratings ($1 \leq$ PR ≤ 3.5) at $F/F_* = \delta/\delta_* = 1$, this principle can be defined as follows:

$$\Delta\mathrm{PR} = \Delta\mathrm{PR}(F/F_*, \delta/\delta_*) \tag{1}$$

where ΔPR is a monotonic function and its minimum is 0 for $F/F_* = \delta/\delta_* = 1$.

The forces felt by a pilot depend on manipulator loading and control sensitivity characteristics, and variables F/F_*, δ/δ_* are the functions of these characteristics:

$$F/F_* = F/F_*\big(F_{\delta m}, F_r, \dots\big), \qquad \delta/\delta_* = \delta/\delta_*\big(F_{\delta m}, F_r, \dots\big)$$

This principle is based on a generalization of certain inherent qualities of the pilot. Both manipulator forces and displacements applied are of great importance to a pilot while controlling. Because of certain human physiological limitations, values of these characteristics that are either too small or too great somehow do not appeal to pilots. The best pilot ratings are achieved in the case of a certain favorable combination of these displacements and forces.

As far as a particular form of Eq. (1) is concerned, it is shown in Figs. 1 and 2 that the following equation agrees adequately with the available experimental data:

$$\Delta\mathrm{PR}(F/F_*, \delta/\delta_*) = \Delta\mathrm{PR}(F/F_*) + \Delta\mathrm{PR}(\delta/\delta_*) \tag{1'}$$

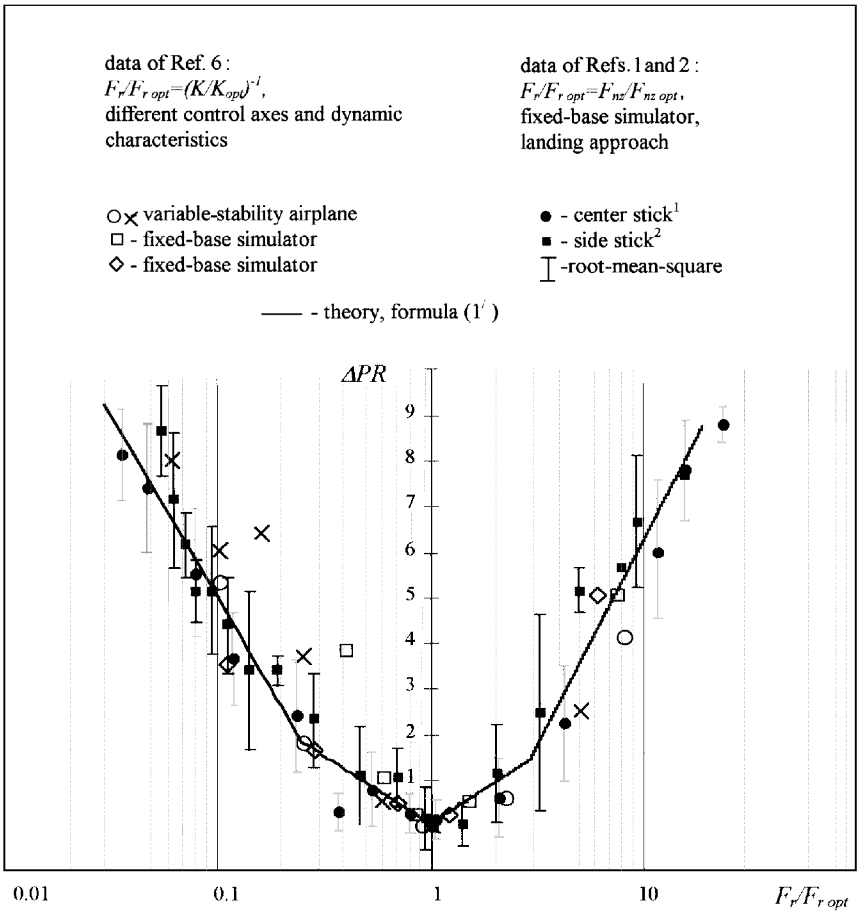

Fig. 1 Pilot rating decrement vs control sensitivity characteristics.

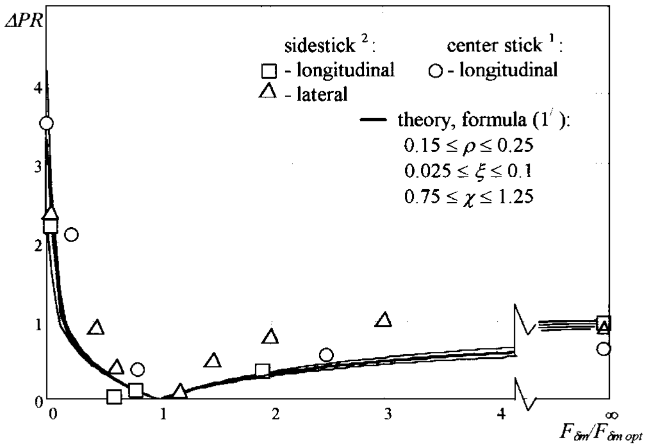

Fig. 2 Pilot ratings vs manipulator force gradient.

where

$$\Delta\mathrm{PR}\left(\frac{F}{F_*}\right) = \begin{cases} -4\log(F/F_*) - 1.5 & \text{at} \quad F/F_* \le 0.25 \\ 1.5\,|\log(F/F_*)| & \text{at} \quad 0.25 < F/F_* < 3 \\ 4.5\log(F/F_*) - 1.4 & \text{at} \quad F/F_* \ge 3 \end{cases}$$

The function $\Delta\mathrm{PR}(\delta/\delta_*)$ takes a similar form.

Note that this particular form of Eq. (1′) is not really important to theoretically substantiate the main results referred to in the present work.

The second principle, or the optimum principle for forces and displacements applied, states that optimum manipulator loading and control sensitivity characteristics are attained if their deviations from the desirable values are minimal.

Essentially, this principle means that a pilot tends to prefer that the combinations of feel-system and control sensitivity characteristics remain around the desirable values of forces and displacements. This can be presented as follows:

$$\min_{F_{\delta m}, F_r, \ldots \in G} J = (F - F_*)^2 + k(\delta - \delta_*)^2 \tag{2}$$

where $F = F(F_{\delta m}, F_r, \ldots)$, $\delta = \delta(F_{\delta m}, F_r, \ldots)$, $F_* = \text{const}$, and $\delta_* = \text{const}$.

The third principle states that, whereas the influence of manipulator and control sensitivity characteristics on handling qualities is being estimated, aircraft state parameters $(n_z, q, p, \dot{p}, \ldots)$ may be assumed independent of manipulator and control sensitivity characteristics $(F_{\delta_m}, F_r, \ldots)$ and aircraft dynamics.

To prove this statement, it is enough to say that approach and landing are performed with about the same accuracy for different aircraft, though differences in their stability and controllability characteristics are considerable. Moreover, this principle is actually used for aircraft controllability analysis, e.g., by McRuer et al.,[6,7] who consider output parameters of a pilot–aircraft closed system independent of control sensitivity characteristics (aircraft gain) if the ranges of variation in these characteristics are narrow. Only pilot ratings depend on these characteristics because a pilot commonly maintains adequate piloting accuracy even when considerable effort is required.

Thus, we have presented the three principles that are the core of our approach: They are considered in greater detail in the next section.

B. Basic Notions in Detail

To estimate the influence of manipulator loading and control sensitivity characteristics on handling qualities according to our principles, we must express the forces F and displacements δ in Eqs. (1) and (2) as functions of manipulator loading and control sensitivity characteristics. There are a few ways to do this.[1–3] We consider one of them.

We assume that control-surface deflection is proportionate to manipulator displacement and that aircraft dynamics are defined by linear differential equations. We also assume that manipulator displacements are sinusoidal (aircraft motion is sinusoidal as well):

$$\delta_m(t) = A_{\delta_m} \sin\omega_* t, \qquad c(t) = A_* \sin(\omega_* t + \varphi) \tag{3}$$

In accordance with the third principle, characteristic frequency ω_* and amplitude A_* do not depend on aircraft controllability characteristics, that is, ω_* and $A_* = \text{const}$. The type of controlled parameter $c(t)$ is determined by the control loop and piloting task considered. For example, as shown in Refs. 2 and 3, to estimate the longitudinal handling qualities for the traditional flight mode, a parameter similar to the one used in the C^* criterion would be appropriate:

$$c(t) = \Delta n_z + (V_0/g)q$$

To define control sensitivity, different parameters, such as F_{nz}, L_{δ_a}, and K, are used. For the sake of uniformity, we use parameter F_r (where $r = n_z, p, \dot{p}, \ldots$), which characterizes force control sensitivity $F_r = F_{nz}, F_{\delta_m}/L_{\delta_a}, F_{\delta_m}/K, \ldots$.

Let us consider the following function of manipulator forces of displacements:

$$F_m(\delta) = F_{\delta_m}\delta_m + F_{\mathrm{br}}\,\mathrm{sgn}(\delta_m) + F_{\mathrm{fr}}\,\mathrm{sgn}(\dot{\delta}_m)$$

A pilot's manipulation of a fixed control stick may produce the same sensation as manipulation of a moveable stick because of the inevitable contraction of his/her muscles. Let us assume that these phantom displacements are proportionate to the forces applied dF_m and felt displacements δ_v are the sum of actual δ_m and phantom manipulator displacements. Similarly, even if the loading characteristics are zero, a pilot deflecting a manipulator inevitably applies some force, e.g., due to hand inertia. Let us assume that this force is proportionate to manipulator displacement $(f\delta_m)$. Thus, we have

$$\delta_v = \delta_m + dF_m, \qquad F_v = F_m + f\delta_m \tag{4}$$

Characteristic values of forces and displacements are assumed maximum in magnitude for sinusoidal manipulator movements:

$$F = \max F_v(t), \qquad \delta = \max \delta_v(t)$$

Substituting Eq. (4) for F and δ, we obtain

$$\delta = \max\delta_m + d\left(F_{\delta_m}\max\delta_m + F_{\mathrm{br}} + F_{\mathrm{fr}}\right)$$

$$F = F_{\delta_m}\max\delta_m + F_{\mathrm{br}} + F_{\mathrm{fr}} + f\max\delta_m$$

From Eq. (3), we have

$$\max\delta_m = A_{\delta_m}, \qquad \frac{A_*}{A_{\delta_m}} = \frac{\left|Y_{c/\delta_m}(j\omega_*)\right|}{F_r\big/F_{\delta_m}}$$

and, thus,

$$\max\delta_m = A_*\frac{F_r\big/F_{\delta_m}}{\left|Y_{c/\delta_m}(j\omega_*)\right|}$$

where Y_{c/δ_m} is the aircraft transfer function in terms of controlled parameter c at $F_r/F_{\delta_m} = 1$. (This function obviously depends on aircraft dynamic characteristics.)

From our preceding statements, the following expressions for characteristic values of forces and displacements can be obtained:

$$\begin{aligned} \delta &= A_*\frac{F_r\big/F_{\delta_m}}{\left|Y_{c/\delta_m}(j\omega_*)\right|} + dA_*\frac{F_r}{\left|Y_{c/\delta_m}(j\omega_*)\right|} + d(F_{\mathrm{br}} + F_{\mathrm{fr}}) \\ F &= \left(F_{\delta_m} + f\right)A_*\frac{F_r/F_{\delta_m}}{\left|Y_{c/\delta_m}(j\omega_*)\right|} + F_{\mathrm{br}} + F_{\mathrm{fr}} \end{aligned} \tag{5}$$

Thus, Eqs. (1), (2), and (5) fully define the influence of loading and control sensitivity characteristics on handling qualities for different aircraft dynamic characteristics.

III. Estimation of Similarity Criteria

To estimate optimum values of manipulator-loading and control-sensitivity characteristics according to Eqs. (1), (2), and (5), we have to know the values of F_*, δ_*, k, d, f, A_*, and ω_* and the form that the transfer function Y_{c/δ_m} takes. Parameters F_*, δ_*, k, d, and f depend only on the manipulator type, whereas constant A_* and characteristic frequency ω_* depend on the aircraft class and the control axes; transfer function Y_{c/δ_m} depends on the control axes involved, on aircraft dynamic performance, and, finally, on the piloting task in question because, in accordance with the type of piloting task, a certain aircraft state parameter c is controlled. All of these parameters can be determined experimentally. They have been determined for class III aircraft with different types of manipulators.[1−3] To determine these parameters for other types of aircraft, special experiments are needed. In this section, a method is proposed to estimate the influence of loading and control sensitivity characteristics without full information about these parameters.

The present part of the work aims at 1) defining the similarity criteria to compare the influence of manipulator loading and control sensitivity characteristics on handling qualities of different aircraft; 2) substantiating theoretically the coincidence of pilot ratings related to nondimensional manipulator loading and control sensitivity characteristics, e.g., $\Delta\mathrm{PR}(F_r/F_{r\,\mathrm{opt}})$ and $F_{r\,\mathrm{opt}}(F_{\delta_m}/F_{\delta_m\,\mathrm{opt}})$, for different aircraft, manipulators, and control axes; and 3) corroborating the theoretical conclusion concerning the similarity of the correlations, thus validating the theoretical approach applied by comparing the experimental data for different conditions (different aircraft, control axes, and manipulators).

Let us introduce the nondimensional parameters for force/displacement gradient, breakout force, friction, control sensitivity, and cost function:

$$\bar{F}_{\delta_m} = \frac{\delta_*}{F_*}F_{\delta_m}, \qquad \bar{F}_{\mathrm{br}} = \frac{F_{\mathrm{br}}}{F_*}, \qquad \bar{F}_{\mathrm{fr}} = \frac{F_{\mathrm{fr}}}{F_*}$$
$$\bar{F}_r = \frac{F_r}{F_*}\frac{A_*}{\left|Y_{c/\delta_m}(j\omega_*)\right|}, \qquad \bar{J} = \frac{J}{F_*^2} \tag{6}$$

With these parameters and Eqs. (5), the initial equations (1) and (2) take the form

$$\Delta\mathrm{PR} = \Delta\mathrm{PR}\left\{\left[\bar{F}_r + \xi\left(\bar{F}_r/\bar{F}_{\delta_m}\right) + \bar{F}_{\mathrm{br}} + \bar{F}_{\mathrm{fr}}\right],\ \left[\left(\bar{F}_r/\bar{F}_{\delta_m}\right) + \rho\left(\bar{F}_r + \bar{F}_{\mathrm{br}} + \bar{F}_{\mathrm{fr}}\right)\right]\right\} \tag{7}$$

$$\min_{\bar{F}_{\delta_m},\bar{F}_r,\ldots\in G} \bar{J} = \left[\bar{F}_r + \xi\left(\bar{F}_r/\bar{F}_{\delta_m}\right) + \bar{F}_{\mathrm{br}} + \bar{F}_{\mathrm{fr}} - 1\right]^2 + \chi\left[\left(\bar{F}_r/\bar{F}_{\delta_m}\right) + \rho\left(\bar{F}_r + \bar{F}_{\mathrm{br}} + \bar{F}_{\mathrm{fr}}\right) - 1\right]^2 \tag{8}$$

where

$$\chi = k(\delta_*/F_*)^2, \qquad \rho = dF_*/\delta_*, \qquad \xi = f\delta_*/F_* \tag{9}$$

Equation (1′) takes the form

$$\Delta\mathrm{PR}(\bar{F},\bar{\delta}) = \Delta\mathrm{PR}(\bar{F}) + \Delta\mathrm{PR}(\bar{\delta}) \tag{7'}$$

where

$$\bar{F} = \bar{F}_r + \xi\left(\bar{F}_r/\bar{F}_{\delta_m}\right) + \bar{F}_{\mathrm{br}} + \bar{F}_{\mathrm{fr}}$$
$$\bar{\delta} = \left(\bar{F}_r/\bar{F}_{\delta_m}\right) + \rho\left(\bar{F}_r + \bar{F}_{\mathrm{br}} + \bar{F}_{\mathrm{fr}}\right)$$

$$\Delta\mathrm{PR}(\bar{F}) = \begin{cases} -4\log\bar{F} - 1.5 & \text{at} \quad \bar{F} \le 0.25 \\ 1.5|\log\bar{F}| & \text{at} \quad 0.25 < \bar{F} < 3 \\ 4.5\log\bar{F} - 1.4 & \text{at} \quad \bar{F} \ge 3 \end{cases}$$

The function $\Delta\mathrm{PR}(\bar{\delta})$ takes a similar form.

In similarity theory, nondimensional coefficients in equations defining a certain phenomenon in terms of nondimensional variables are called similarity criteria. In our case, the similarity criteria are χ, ρ, and ξ.

In many cases, the similarity-criteria approach simplifies the investigation of manipulator loading and control sensitivity influence on handling qualities; namely, it allows us to define handling qualities even if the values of the relevant parameters are unknown or the information is incomplete. This follows from the fact that only three similarity criteria are included in Eqs. (7), (7′), and (8), which means that five parameters (F_*, δ_*, k, d, f), not eight, are sufficient for our purposes. We can do without the values of parameters A_*, ω_*, and the transfer function Y_{c/δ_m} because they are not included in Eqs. (7–9).

The similarity-criteria approach allows us to use simple generalization rules for different types of correlations between pilot ratings, manipulator loading, and control sensitivity characteristics. For example, a function of nondimensional variables is the same for various conditions provided that 1) the values of the similarity criteria are equal (or their influence is negligible) and 2) similarity criteria are not included in the function in question.

It is impossible to consider all of the cases in which the similarity-criteria approach is applicable. We illustrate its applicability to the correlations considered and demonstrate the validity of the approach by comparing the estimated and experimental data. Let us consider three examples that are important in practice. We assume that

$$\bar{F}_{\mathrm{br}} = \bar{F}_{\mathrm{fr}} = 0 \tag{10}$$

A. Example 1: Influence of Control Sensitivity Characteristics on Handling Qualities

We demonstrate that the pilot-ratings function of nondimensional sensitivity characteristics $F_r/F_{r\,\mathrm{opt}}$ is roughly the same for different aircraft, manipulators, and control axes, provided that

$$\bar{F}_{\delta_m} = \bar{F}_{\delta_m\,\mathrm{opt}}\Big|_{\bar{F}_r=\bar{F}_{r\,\mathrm{opt}}} \tag{11}$$

To prove our statement theoretically, we first mention that

$$\bar{F}_r = \frac{\bar{F}_r}{\bar{F}_{r\,\mathrm{opt}}}\bar{F}_{r\,\mathrm{opt}} = \frac{F_r}{F_{r\,\mathrm{opt}}}\bar{F}_{r\,\mathrm{opt}}$$

Thus, if Eq. (11) is true, Eq. (7) takes the following form:

$$\Delta\mathrm{PR} = \Delta\mathrm{PR}\left(\frac{F_r}{F_{r\,\mathrm{opt}}}\bar{F}_{r\,\mathrm{opt}} + \xi\frac{F_r}{F_{r\,\mathrm{opt}}}\frac{\bar{F}_{r\,\mathrm{opt}}}{\bar{F}_{\delta_m\,\mathrm{opt}}},\ \frac{F_r}{F_{r\,\mathrm{opt}}}\frac{\bar{F}_{r\,\mathrm{opt}}}{\bar{F}_{\delta_m\,\mathrm{opt}}} + \rho\frac{F_r}{F_{r\,\mathrm{opt}}}\bar{F}_{r\,\mathrm{opt}}\right) \tag{12}$$

From Eq. (8) (if we assume, for example, that $\bar{J} = 0$), we receive

$$\bar{F}_{r\,\mathrm{opt}} = (1-\xi)/(1-\rho\xi), \qquad \bar{F}_{\delta_m\,\mathrm{opt}} = (1-\xi)/(1-\rho) \tag{13}$$

Substituting Eqs. (13) into Eq. (12), we arrive at the function to be found:

$$\Delta\mathrm{PR} = \Delta\mathrm{PR}\left(\frac{F_r}{F_{r\,\mathrm{opt}}}\right)$$

It is seen that there are no similarity criteria in the function in question. This means that the form of the function remains the same for different aircraft, manipulators, and control axes, which was to be proved.

For the general case, the character of the function in question has been revealed by McRuer and Jex[6] in the course of experiments. Their data are presented in Fig. 1, together with our experimental data on pilot ratings for longitudinal control sensitivity.[1−3] The latter were obtained on a fixed-base simulator while modeling landing approaches for aircraft with side and center sticks; loading characteristics were in agreement with Eqs. (10) and (11); and aircraft dynamic characteristics for $F_{r\,\mathrm{opt}}$ corresponded to PR = 1–1.5. The data compared demonstrate in all cases the similarity of pilot-rating/sensitivity-characteristic correlations regardless of the differences in aircraft type and dynamic characteristics, manipulators, control axes, and control sensitivity characteristics. Because of its general character, the function in question is widely used in handling-qualities studies. For example, McRuer et al.[7] applied it in their work on pilot induced oscillations (PIO), but there had not been any theoretical approach developed to define this function, as the authors mentioned.

The example presented substantiates theoretically the general character of the functions considered. The conditions under which the functions are compared are as follows. Conditions of Eqs. (10) and (11) must be satisfied, and pilot ratings of the aircraft compared must remain approximately the same for $F_{\delta_m \text{opt}}$ and $F_{r\,\text{opt}}$.

We did not use a particular form of Eq. (1) as an argument to prove the general character of the function considered, but, to demonstrate that Eq. (1′) is a particular form of Eq. (1), in Fig. 1 we show the curve received from Eq. (7′).

Thus, the example shows the general character of the function $\Delta\text{PR} = \Delta\text{PR}(F_r/F_{r\,\text{opt}})$, and we can define its forms on the basis of the proposed theory, even if the values of the parameters in Eqs. (1), (2), and (5) are unknown. Satisfactory agreement of the estimated and experimental data proves the validity of the proposed approach.

B. Example 2: Influence of Loading Gradient on Handling Qualities

We demonstrate that the pilot-ratings function of the loading gradient referred to its optimum value, i.e., $\Delta\text{PR}(F_{\delta_m}/F_{\delta_m \text{opt}})$, remains roughly the same for different manipulators, aircraft, dynamic characteristics, and piloting tasks, provided that control sensitivity remains optimum for each loading gradient value:

$$\bar{F}_r = \bar{F}_{r\,\text{opt}}\left(\frac{F_{\delta_m}}{F_{\delta_m \text{opt}}}\right) \tag{14}$$

In Fig. 2 the experimental data are shown for center and side sticks. The data were received for approach-mode simulation on a fixed-base flight simulator. The experimental procedure is described in Ref. 2.

All of the data corroborate our theoretical conclusion that the qualitative correlation between pilot ratings and loading gradient remains roughly the same for various manipulators if Eqs. (10) and (14) are true.

C. Example 3: Influence of Loading Gradient on Optimum Control Sensitivity Characteristics

The influence of loading gradient on handling qualities is known to be similar to that of control sensitivity. A number of their concurrent influence regularities are mentioned in Ref. 4, e.g., for their important roles in the study of the PIO phenomenon and modeling, but these regularities are not strictly defined. Our theoretical approach allows us to theoretically substantiate the regularities in question and to define them quantitatively. We demonstrate the validity of the proposed approach for the function of optimum values

$$\frac{F_{r\,\text{opt}}}{F_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}}} \quad \text{of} \quad \frac{F_{\delta_m}}{F_{\delta_m \text{opt}}}$$

Let us define this function and show its similarity for various manipulator types and aircraft. This function can be derived easily from Eq. (8) (when $\partial\bar{J}/\partial\bar{F}_r = 0$), if Eqs. (15) and (16) are true. It takes the form

$$\frac{F_{r\,\text{opt}}}{F_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}}} = \frac{F_{\delta_m}}{F_{\delta_m \text{opt}}}(1-\xi\rho)\frac{\left(F_{\delta_m}/F_{\delta_m \text{opt}}\right)(1-\xi)+\xi(1-\rho)+\chi\left[1-\rho+\left(F_{\delta_m}/F_{\delta_m \text{opt}}\right)\rho(1-\xi)\right]}{\left[\left(F_{\delta_m}/F_{\delta_m \text{opt}}\right)(1-\xi)+\xi(1-\rho)\right]^2+\chi\left[1-\rho+\left(F_{\delta_m}/F_{\delta_m \text{opt}}\right)\rho(1-\xi)\right]^2} \tag{17}$$

The form of the function considered (and all other functions of nondimensional characteristics) is the same for different aircraft, dynamic characteristics, and piloting tasks because all three similarity criteria in Eq. (9) contain the parameters that depend only on the type of the manipulator.

From Refs. 1–3, it follows that average values of the similarity criteria for various manipulators are $\chi \approx 1$, $\rho \approx 0.2$, and $\xi \approx 0.05$. Their deviations from these values are about 25–100%. To prove the similarity of the function in question for different manipulator types, it is sufficient to demonstrate that no deviation of the similarity criteria from the values mentioned earlier affects the correlation in question to any considerable extent, which is shown in Fig. 2; the curves are received from Eq. (7′) for different values of similarity criteria and condition equations (10) and (14).

For simplicity, no particular expression for the function $\Delta\text{PR} \times (F_{\delta_m}/F_{\delta_m \text{opt}})$ is presented here but it can be derived from Eq. (7′) under two conditions. First, the following expressions are substituted for $\bar{F}_{\delta_m}$:

$$\bar{F}_{\delta_m} = \frac{F_{\delta_m}}{F_{\delta_m \text{opt}}}\frac{1-\xi}{1-\rho} \tag{15}$$

Equation (15) follows from

$$\bar{F}_{\delta_m} = \frac{\bar{F}_{\delta_m}}{\bar{F}_{\delta_m \text{opt}}}\bar{F}_{\delta_m \text{opt}} = \frac{F_{\delta_m}}{F_{\delta_m \text{opt}}}\bar{F}_{\delta_m \text{opt}}$$

if Eq. (13) is true. Second, the expression

$$\bar{F}_r = \bar{F}_{r\,\text{opt}} = \frac{\bar{F}_{r\,\text{opt}}}{\bar{F}_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}}}\bar{F}_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}}$$

$$= \frac{F_{r\,\text{opt}}}{F_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}}}\bar{F}_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}} \tag{16}$$

is substituted for $\bar{F}_r$ in Eq. (7′), where $\bar{F}_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}}$ is determined from Eq. (13) and $F_{r\,\text{opt}}/(F_{r\,\text{opt}}|_{F_{\delta_m}=F_{\delta_m \text{opt}}})$ is determined from Eq. (17) in example 3.

The curves presented in Fig. 3 are generated for different similarity-criteria values. They demonstrate that the differences in similarity-criteria values do not produce any considerable effect on the form of the function. This proves its general character for different manipulator types and aircraft characteristics.

The function in question can be used in practical work to ensure maximum similarity of handling qualities modeled on simulators if their loading gradients differ from those of the aircraft simulated.

Let us assume that an aircraft has optimum values of control sensitivity F_{r1} and loading gradient $F_{\delta_m 1}$. They correspond to the point (1,1) of the curve in Fig. 3. We consider the simulated environment to be ideal except for the loading-gradient value $F_{\delta_m 2} \neq F_{\delta_m 1}$. It follows that the maximum similarity of handling qualities is achieved if the simulator control sensitivity F_{r2} is adjusted according to the following expression:

$$\frac{F_{r2}}{F_{r1}} = \frac{F_{r2\,\text{opt}}\left(F_{\delta_m 2}/F_{\delta_m \text{opt}}\right)}{F_{r1\,\text{opt}}\left(F_{\delta_m 1}/F_{\delta_m \text{opt}}\right)} = \frac{F_{r2\,\text{opt}}\left(F_{\delta_m 2}/F_{\delta_m 1}\right)}{F_{r1\,\text{opt}}|_{F_{\delta_m 1}=F_{\delta_m \text{opt}}}}$$

where the latter term is defined from Eq. (17) or from the curve in Fig. 3.

In Fig. 3 the experimental data for center and side sticks from Refs. 1–3 are shown. The agreement between the experimental data and estimations proves the validity of our theoretical approach.

IV. Requirements for Dynamic Characteristics of Feel Systems

The required characteristics of various feel systems are considered in a number of publications.[8,9] Among them, the dynamic characteristics are the least developed.

The need to meet very strict requirements for dynamic accuracy while reproducing loading characteristics is determined by the capabilities of the pilot. It is well known that pilots deliberately operate at the frequencies up to 1–1.5 Hz, but, in many cases, higher frequencies in the limb-manipulator system may influence piloting. For example, high-frequency oscillations up to 3 Hz may occur on modern aircraft with certain structural elasticity, manipulator characteristics, role-mode time constants, etc.[10,11] Moreover, in some cases

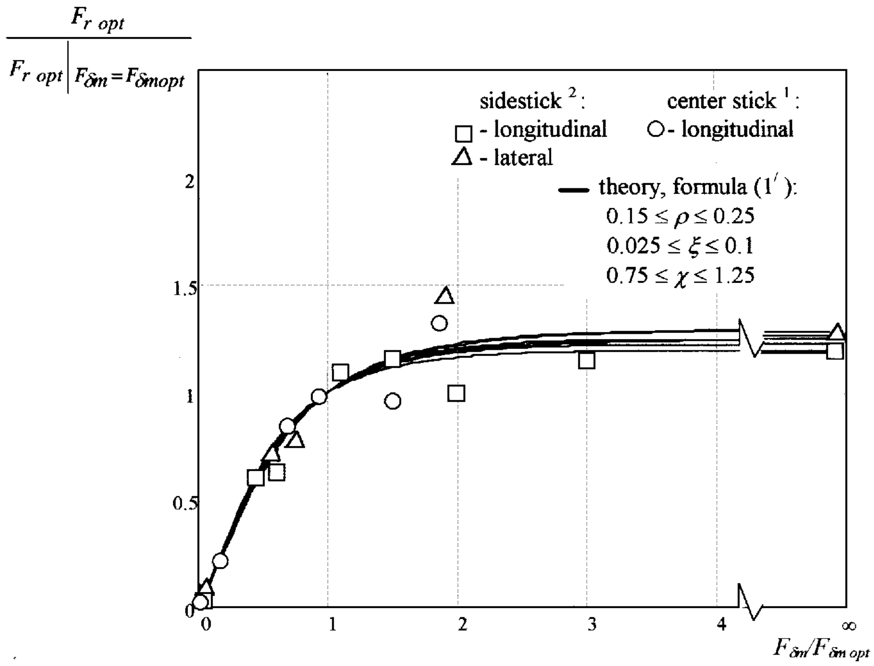

Fig. 3 Optimum control sensitivity characteristics vs manipulator force gradient.

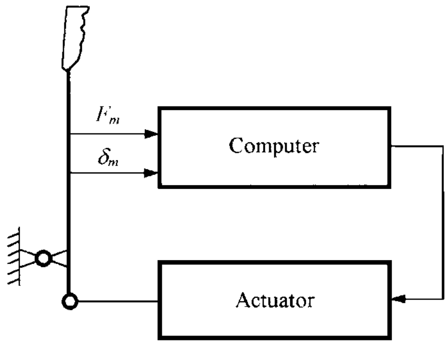

Fig. 4 Structure of manipulator feel system.

the spectrum of pilot control movements contains components of higher frequency. In our experiments the frequency limits of pilot control movements were up to 2–3 Hz in yaw, 5 Hz in pitch, and 7 Hz in roll. It proves the need for the feel system in a research simulator to be able to reproduce the frequencies within this range without noticeable dynamic distortions.

The selection of dynamic characteristics of feel-system elements (actuator and computer) also is determined by feel-system stability requirements. The possibility of high-frequency oscillations (15 Hz and over) is one of the factors affecting the selection of dynamic characteristics. These oscillations are due to the force transducer sensing the feel-system elastic mode; such a case was considered in Refs. 8 and 9.

Another factor affecting system stability and thus its dynamic characteristics is the need to reproduce great loading gradients, e.g., for imitating breakout force. The role of this factor in maintaining system stability depends on the feel-system structure. Let us consider the most widely used feel-system structure, shown in Fig. 4. It suggests that the actuator is an ideal integrator and the computer has no time delay. However, in practice, the dynamics of an actuator at high frequencies differ from ideal dynamics. To estimate the required actuator dynamics, we assume that they are described by the function

$$Y_a = \frac{1}{s}\frac{1}{(T_a s + 1)}$$

Because of quantization, computer performance differs from the ideal as well. Its transfer function is

$$Y_{\text{cp}} = e^{-T_c s}\frac{1 - e^{-T_c s}}{T_c s}$$

where $e^{-T_c s}$ is the factor of time delay introduced by the computer and $(1 - e^{-T_c s})/T_c s$ is the transfer function of A/D converters.

Let us assume that we have to reproduce the following loading function:

$$m\ddot{\delta}_m + F_{\dot{\delta}_m}\dot{\delta}_m + F_{\delta_m}\delta_m = F_m$$

The task is to estimate, within the given stability limitations, the permissible values of T_a and T_c that allow the predetermined values of m, F_{δ_m}, and $F_{\dot{\delta}_m}$ to be reproduced.

It can be shown that, in terms of stability, the T_a and T_c requirements are strictest when m is minimum and the gradient F_{δ_m} is maximum. The data for different manipulators show that the approximate minimum values are $m = 2$ kg and $F_{\dot{\delta}_m} = 80\,\text{N}\cdot(\text{m/s})^{-1}$. The value of the force gradient for imitating the breakout force is no less than $F_{\delta_m} = 30$–100 kN/m, according to Ref. 8.

It can be shown that, for these parameters, a feel system is stable if the sum of T_a and $3/2T_c$ does not exceed 5–10 ms.

We have considered just a common type of feel-system structure 1) to demonstrate the method for estimating the dynamic characteristics required of an actuator and a computer and 2) to show how strict these requirements are. Requirements may be less strict for some simulated loading laws, for certain system-element characteristics, or when corrective filters are used.

V. Conclusions

A previously developed theoretical approach and a similarity theory were applied to estimate the influence of manipulator loading and control sensitivity characteristics on aircraft handling qualities. Similarity criteria for loading and control sensitivity characteristics were proposed and substantiated. Agreement was demonstrated between experimental data and estimations based on the proposed approach. Some data were presented to substantiate the requirements for feel-system dynamic characteristics of flight simulators.

The proposed methodology (theoretical approach and similarity criteria) solves various problems of theoretical and empirical studies of aircraft handling qualities; its validity was shown for a number of examples.

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